Phys 313 Lecture 4

Friday, September 4, 2026 3:52 PM

Electric Charge and Electric Field

Electric charge → Two Flavors

Chocolate + Vanilla

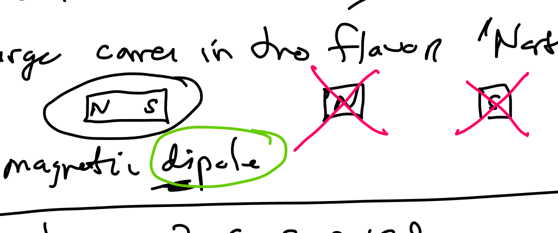
Plus Minus → Equal amounts of + and − charge
charge neutral

Monopole → Scalar

Gravitational Charge Mass $\downarrow m \vec{g}$ $\vec{F} = -\frac{G m_1 m_2}{r^2}\hat{r}$

positive

Magnetic charge comes in two flavors "North", "South"



magnetic dipole

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① Electric charge is conserved.

Non-relativistic, slow speed

non-QED

Conservation is both Global and Local

Local conservation

Continuity Equation

$$\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$$

current density — Local charge density

② Charge is quantized in units of the electron charge.

$e = 1.602\times10^{-19}\,C$

Charge on object $Q = \pm N e$ Free charges.

$N = 0, 1, 2, 3, \ldots$

Fractional charge Quarks $\pm\frac{e}{3}, \pm\frac{2}{3}e$

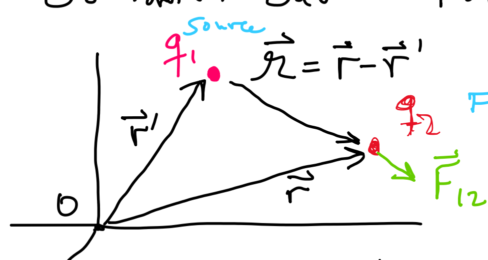
in nucleus

Fractional Quantum Hall effect $\frac{2}{9}e$

Condensed matter system

Coulomb's Law

Force between two charges.



q_1, q_2 can be ±

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{\mathscr{r}}|^2}\hat{\mathscr{r}}$$

unit separation vector

No dimensions

→ $q_1, q_2 > 0$ or < 0 Repulsive force

→ q_1, q_2 different signs Attractive force

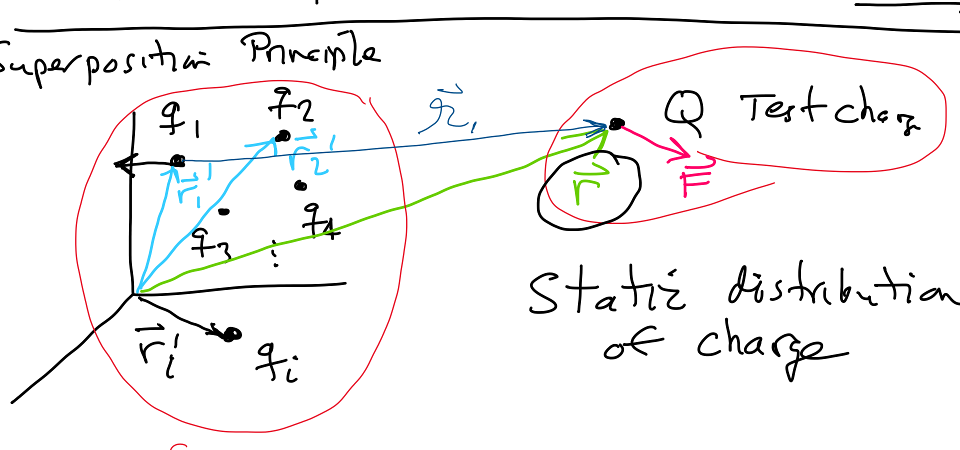
Linear in charge q_1, in charge q_2

→ $\varepsilon_0 = 8.85\times10^{-12}\,\frac{C^2}{Nm^2} = 8.85\,\frac{pF}{m}$

permittivity of free space

$\vec{E}$

Superposition Principle



Static distribution of charge

Source

$$\vec{F}_Q = \vec{F}_{1Q} + \vec{F}_{2Q} + \cdots \vec{F}_{iQ} + \cdots \vec{F}_{NQ}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 Q}{|\vec{\mathscr{r}}_1|^2}\hat{\mathscr{r}}_1 + \cdots \frac{q_i Q}{|\vec{\mathscr{r}}_i|^2}\hat{\mathscr{r}}_i + \cdots \frac{q_N Q}{|\vec{\mathscr{r}}_N|^2}\hat{\mathscr{r}}_N\right)$$

$$\vec{F}_Q = Q\underbrace{\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1\hat{\mathscr{r}}_1}{|\vec{\mathscr{r}}_1|^2} + \cdots \frac{q_N\hat{\mathscr{r}}_N}{|\vec{\mathscr{r}}_N|^2}\right)}_{\vec{E}(\vec{r})}$$

Collection of 2-body interactions. The presence of a third ~~the~~ charge has no effect.

Electric Field at the location of charge Q.

$\vec{E}(\vec{r}) = \frac{\vec{F}_Q}{Q}$ Electric field.

field point

$\vec{F}_Q = Q\vec{E}(\vec{r})$

$\vec{E}(\vec{r}) = \lim_{Q\to0}\left(\frac{\vec{F}_Q}{Q}\right)$ limiting process

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i\hat{\mathscr{r}}_i}{|\vec{\mathscr{r}}_i|^2}$$

Electric field at the field point.

linear superposition

Continuous Charge Distributions

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int dq\,\frac{\hat{\mathscr{r}}}{|\vec{\mathscr{r}}|^2}$$

$dq \to \begin{cases}\text{line charges}\\ \text{surface charge}\\ \text{volume charge}\end{cases}$

①

Define linear charge density $\lambda = \frac{\text{charge}}{\text{unit length}}$

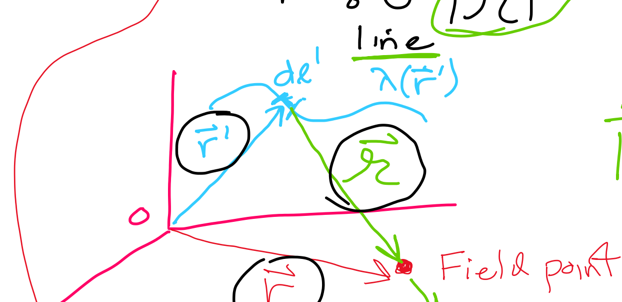
Ex $\lambda = 5\,\frac{\mu C}{m}$ $\lambda = \lambda(\ell)$ location ℓ

$dq = \lambda\, d\ell'$ $d\ell'$

• Field point

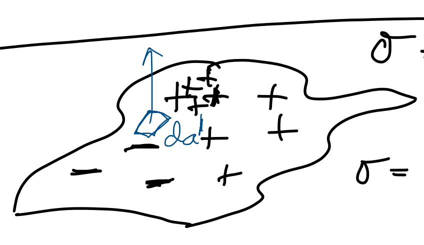
$$E(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{\text{line}}\frac{\lambda(\vec{r}\,')}{|\vec{\mathscr{r}}|^2}\hat{\mathscr{r}}\,d\ell'$$

$\frac{\vec{\mathscr{r}}}{|\vec{\mathscr{r}}|} = \hat{\mathscr{r}}$



Surface Charge

$\sigma = \frac{\text{charge}}{\text{unit area}}$



$\sigma = \sigma(\vec{r}\,')$

$dq = \sigma\, da'$

$= \sigma(\vec{r}\,')\, da'$

$$\boxed{\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\iint_{\text{surface}}\frac{\sigma(\vec{r}\,')}{|\vec{\mathscr{r}}|^2}\hat{\mathscr{r}}\,da'}$$

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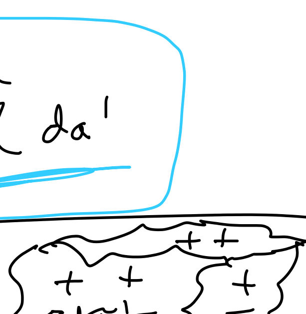
volume charge

$\rho = \frac{\text{charge}}{\text{volume}}$ Ex $\rho = -\frac{3.9\,\mu C}{m^3}$



$\rho(\vec{r}\,')$

$dq = \rho\, d\tau'$

→ $$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\iiint_{\text{Volume}}\frac{\rho(\vec{r}\,')}{|\vec{\mathscr{r}}|^2}\hat{\mathscr{r}}\,d\tau'$$

Question: $\vec{E}(\vec{r})$: Is there any information in $\vec{E}(\vec{r})$ about the source charge that created the electric field?

No and Yes Gauss' Law

$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$ ← Volume charge density

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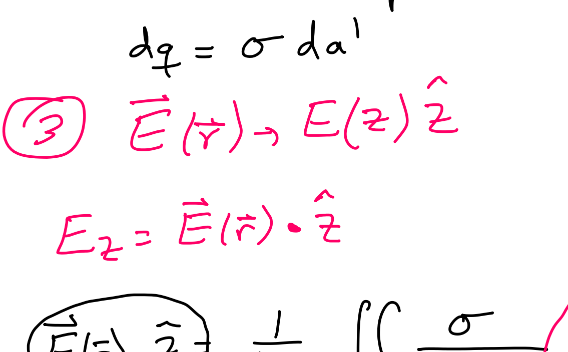
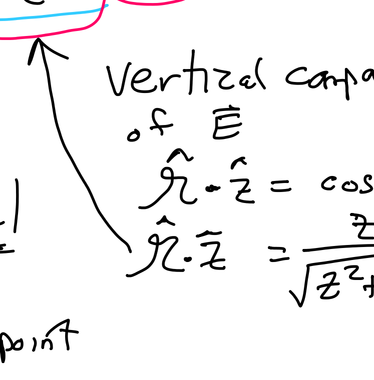
Problem 2.6 in Griffiths



uniformly charge circular disk

① Cylindrical Coordinate System

Cylindrical Symmetry



$\Rightarrow \vec{E}(\vec{r}) = E(z)\hat{z}$

② choose an origin

$dq = \sigma\, da'$

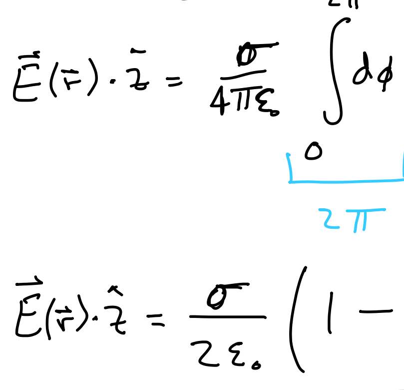
③ $\vec{E}(\vec{r}) \to E(z)\hat{z}$

$E_z = \vec{E}(\vec{r})\cdot\hat{z}$

$$\vec{E}(\vec{r})\cdot\hat{z} = \frac{1}{4\pi\varepsilon_0}\iint_{\text{Disk}}\frac{\sigma}{|\vec{\mathscr{r}}|^2}\hat{\mathscr{r}}\cdot\hat{z}\,da'$$

$da' = s\,ds\,d\phi$

Vertical component of $\vec{E}$



$\sqrt{z^2+s^2} = |\vec{\mathscr{r}}|$

$\hat{\mathscr{r}}\cdot\hat{z} = \cos\theta$

$\hat{\mathscr{r}}\cdot\hat{z} = \frac{z}{\sqrt{z^2+s^2}}$

$$\vec{E}(\vec{r})\cdot\hat{z} = \frac{\sigma}{4\pi\varepsilon_0}\underbrace{\int_0^{2\pi}d\phi}_{2\pi}\int_0^R s\,ds\,\underbrace{\frac{1}{z^2+s^2}\frac{z}{\sqrt{z^2+s^2}}}_{\text{No }\phi\text{-dependence}}$$

$$\vec{E}(\vec{r})\cdot\hat{z} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{z}$$

Question:

① What happens as $R\to\infty$ Infinite disk.

$\vec{E}(\vec{r})\cdot\hat{z} = \frac{\sigma}{2\varepsilon_0}\hat{z}$ Infinite sheet charge result

② What happens as $z\to\infty$ $z \gg R$

$\vec{E}(\vec{r})\cdot\hat{z} \to \frac{Q}{4\pi\varepsilon_0 z^2}\hat{z}$ $Q \equiv \sigma\underbrace{\pi R^2}_{\text{area of the disk}}$

Coulomb's law